Postulates of Quantum-Mechanics

The classical mechanics holds good to explain the all phenomenon and properties of Macroscopic particles but it fails to explain microscopic world like electron, atom ... etc.

Hence a new Concept like wave mechanical treatment is required to explain all phenomenon and properties of microscopic world. Now a new set of rules come in to existance to explain it; which is known as Postulates of quantum-mechanics.

Postulates for a system moving in one dimension i.e along X-co-ordinate are given below.

Postulate - 1. The Physical state of a system at time t is described by the wave function $\psi(x,t)$.

2. The wave function $\psi(x,t)$ and its first and second derivatives $\frac{\partial \psi(x,t)}{\partial x}$ and $\frac{\partial^2 \psi(x,t)}{\partial x^2}$ are continuous, finite and single valued for all the value of x.

And also the wave function $\psi(x,t)$ is normalised i.e $\int_{-\infty}^{+\infty} \psi^*(x,t)\,\psi(x,t)\,dx = 1$

3. A Physically observable quantity can be represented by a Hermitian Operator.

Operator $\hat{A}$ is said to be Hermitian if it satisfies the condition given below.

$$\int \psi_i^* \hat{A} \psi_j \, dx = \int \psi_j (\hat{A}\psi_i)^* \, dx$$

where ψ_i and ψ_j are the wavefunctions representing the physical states of the quantum system like atom or molecule.

4. The allowed values of an observable A are the eigen values.

$$\hat{A}\psi_i = a_i \psi_i$$

Here $\hat{A}$ is the operator for the observable and ψ_i is an eigenfunction of $\hat{A}$ with eigen value a_i. i.e measurement of the observable A yields the eigen value a_i.

5. The average value $\langle A \rangle$ of an observable A corresponding to the operator $\hat{A}$ is obtained from the relation, $\langle A \rangle = \int_{-\infty}^{+\infty} \psi^* \hat{A} \psi \, dx$

where the function ψ is assumed to be normalized, when it fulfill the condition of normalisation.

The average value along x-coordinate is given as

$$\langle x \rangle = \int_{-\infty}^{+\infty} \psi^* \hat{x} \psi \, dx$$

6. The wave function $\psi(x,t)$ is a solution of the time-dependent Schrodinger

$$\hat{H}\psi(x,t) = i\hbar \frac{\partial \psi(x,t)}{\partial t},$$

Postulate – 7 → To every observable such as position, momentum, energy ---- etc. in classical mechanics, there corresponds an operator in quantum mechanics.

Classical - Mechanical observables and their corresponding Quantum Mechanical Operators

Observable		Operator	
Name	Symbol	Symbol	Operation
Position	x	$\hat{x}$	Multiplication by x
	y	$\hat{y}$	Multiplication by y
Momentum	P_x	$\hat{P}_x$	$\frac{h}{2\pi i}\frac{\partial}{\partial x} = -i\hbar\frac{\partial}{\partial x}$
	P	$\hat{P}$	$\frac{h}{2\pi i}\left(i\frac{\partial}{\partial x}+j\frac{\partial}{\partial y}+k\frac{\partial}{\partial z}\right) = -i\hbar\left(i\frac{\partial}{\partial x}+j\frac{\partial}{\partial y}+k\frac{\partial}{\partial z}\right)$
Kinetic Energy	K	$\hat{K}$	$\frac{-h^2}{8\pi^2 m}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right) = \frac{-\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)$
Potential Energy	$V(x)$	$\hat{V}(x)$	Multiplication by $V(x)$
Total Energy	E	$\hat{H}$	$\frac{-h^2}{8\pi^2 m}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right) + V(x,y,z) = -\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)+V(x,y,z)$
Angular Momentum	$L_x = yP_z - zP_y$	$\hat{L}_x$	$-i\hbar\left(y\frac{\partial}{\partial z} - z\frac{\partial}{\partial y}\right)$
	$L_y = zP_x - xP_z$	$\hat{L}_y$	$-i\hbar\left(z\frac{\partial}{\partial x} - x\frac{\partial}{\partial z}\right)$
	$L_z = xP_y - yP_y$	$\hat{L}_z$	$-i\hbar\left(x\frac{\partial}{\partial y} - y\frac{\partial}{\partial x}\right)$